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Generalized Friedberg-Lee model for neutrino masses and leptonic CP violation from μ-τ symmetry breaking

Zhi-zhong Xing, He Zhang and Shun Zhou
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

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Assuming the Majorana nature of massive neutrinos, we generalize the Friedberg-Lee neutrino mass model to include CP violation in the neutrino mass matrix M_ν. The most general case with all the free parameters of M_ν being complex is discussed. We show that a favorable neutrino mixing pattern (with $\theta_{12} \approx 35.3^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} \neq 0^\circ$ and $\delta = 90^\circ$) can naturally be derived from M_ν, if it has an approximate or softly-broken μ-τ symmetry. We also point out a different way to obtain the nearly tri-bimaximal neutrino mixing pattern with $\delta = 0^\circ$ and non-vanishing Majorana phases.

Keywords: neutrino mixing; Friedberg-Lee model; μ-τ symmetry breaking

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Recently, a novel neutrino mass model has been proposed by Friedberg and Lee (FL).[1] The neutrino mass operator in the FL model is simply given by

$$\begin{aligned}\mathcal{L}_{\nu-\rm mass} = {} & a\left(\overline{\nu}_\tau - \overline{\nu}_\mu\right)\left(\nu_\tau - \nu_\mu\right) + b\left(\overline{\nu}_\mu - \overline{\nu}_e\right)\left(\nu_\mu - \nu_e\right) + c\left(\overline{\nu}_e - \overline{\nu}_\tau\right)\left(\nu_e - \nu_\tau\right) \\ & + m_0\left(\overline{\nu}_e\nu_e + \overline{\nu}_\mu\nu_\mu + \overline{\nu}_\tau\nu_\tau\right) \;, \end{aligned} \tag{1}$$

where the parameters a, b, c and m_0 are all assumed to be *real*, and the charged-lepton mass matrix is taken to be diagonal. A salient feature of $\mathcal{L}_{\nu-\rm mass}$ is its partial gauge-like symmetry; i.e., its a, b and c terms are invariant under the transformation $\nu_\alpha \to \nu_\alpha + z$ (for $\alpha = e, \mu, \tau$) with z being a space-time independent constant element of the Grassmann algebra.[1] From Eq. (1), one can directly write down the neutrino mass matrix:

$$M_\nu \;=\; m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \;+\; \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & c+a \end{pmatrix} \;. \tag{2}$$

Two interesting features can be inferred from the diagonalization of M_ν. First, the neutrino mass matrix takes a magic form,[2] in which the sums of rows and columns are all equal to m_0. The unitary matrix used to diagonalize M_ν must have one eigenvector with three equal components $1/\sqrt{3}$. Second, when $b = c$ holds, it is very easy to check that the neutrino mass operator $\mathcal{L}_{\nu-\rm mass}$ has the exact μ-τ symmetry (i.e., $\mathcal{L}_{\nu-\rm mass}$ is invariant under the exchange of μ and τ indices).[3] In

addition, one may consider to remove one degree of freedom from $\mathcal{L}_{\nu-\rm mass}$ or M_ν (for instance, by setting $c = 0$).[1]

To include CP or T violation into the FL model, one may insert the phase factors $e^{\pm i\eta}$ into Eq. (1) by replacing the term $c\left(\overline{\nu}_e - \overline{\nu}_\tau\right)\left(\nu_e - \nu_\tau\right)$ with the term $c\left(e^{-i\eta}\overline{\nu}_e - \overline{\nu}_\tau\right)\left(e^{+i\eta}\nu_e - \nu_\tau\right)$.[1] The resultant neutrino mass matrix is no longer symmetric, hence it describes Dirac neutrinos instead of Majorana neutrinos. However, in most of the realistic models, the Majorana nature is preferable to the Dirac nature of neutrinos. Hence, in this work, we aim to generalize the FL model to include CP and T violation for massive Majorana neutrinos.

Let us start from the generic analysis with all the parameters of M_ν in Eq. (2) being complex. For Majorana neutrinos, M_ν is symmetric and can be diagonalized by the transformation $V^\dagger M_\nu V^* = {\rm Diag}\{m_1, m_2, m_3\}$, in which m_i (for $i = 1, 2, 3$) stand for the neutrino masses. After a straightforward calculation, the neutrino mixing matrix V turns out to be

$$V = \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\frac{\theta}{2} & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\frac{\theta}{2}e^{-i\delta} \\ -\frac{1}{\sqrt{6}}\cos\frac{\theta}{2} - \frac{1}{\sqrt{2}}\sin\frac{\theta}{2}e^{i\delta} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\cos\frac{\theta}{2} - \frac{1}{\sqrt{6}}\sin\frac{\theta}{2}e^{-i\delta} \\ -\frac{1}{\sqrt{6}}\cos\frac{\theta}{2} + \frac{1}{\sqrt{2}}\sin\frac{\theta}{2}e^{i\delta} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\cos\frac{\theta}{2} - \frac{1}{\sqrt{6}}\sin\frac{\theta}{2}e^{-i\delta} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{3}$$

where the explicit expressions of θ and δ are

$$\tan\theta = \frac{\sqrt{X^2 + Y^2}}{Z} \; , \qquad \tan\delta = \frac{X}{Y} \; , \tag{4}$$

and the definitions of X, Y and Z can be found in Ref. 4. Furthermore, three mass eigenvalues of M_ν and two Majorana phases of V are found to be

$$\begin{aligned} m_1 &= \left| T_1 \cos^2\frac{\theta}{2} - T_2 \sin\theta e^{-i\delta} + T_3 \sin^2\frac{\theta}{2} e^{-i2\delta} \right| \; , \qquad m_2 = |m_0| \; , \\ m_3 &= \left| T_3 \cos^2\frac{\theta}{2} + T_2 \sin\theta e^{+i\delta} + T_1 \sin^2\frac{\theta}{2} e^{+i2\delta} \right| \; ; \end{aligned} \tag{5}$$

and

$$\begin{aligned} \rho &= \frac{1}{2}\arg\left(\frac{T_1 \cos^2\frac{\theta}{2} - T_2 \sin\theta e^{-i\delta} + T_3 \sin^2\frac{\theta}{2} e^{-i2\delta}}{T_3 \cos^2\frac{\theta}{2} + T_2 \sin\theta e^{+i\delta} + T_1 \sin^2\frac{\theta}{2} e^{+i2\delta}} \right) \; , \\ \sigma &= \frac{1}{2}\arg\left(\frac{m_0}{T_3 \cos^2\frac{\theta}{2} + T_2 \sin\theta e^{+i\delta} + T_1 \sin^2\frac{\theta}{2} e^{+i2\delta}} \right) \; . \end{aligned} \tag{6}$$

Again, the expressions of T_i have also been listed in Ref. 4.

We proceed to consider two special but interesting scenarios of the generalized FL model and explore their respective consequences on three neutrino mixing angles and three CP-violating phases.

Scenario (A): a and m_0 are real, and $b = c^*$ are complex. Note that the μ-τ symmetry of M_ν is softly broken in this case, because $|b| = |c|$ holds. By using the generic results given in Eqs. (3)-(6), one can easily arrive at $\tan\theta = \sqrt{3}\ \mathrm{Im}\,(b)/\left[m_0 + a + 2\mathrm{Re}\,(b)\right]$, $\delta = 90^\circ$,

$$
\begin{aligned}
m_1 &= \sqrt{\left[m_0 + a + 2\mathrm{Re}\,(b)\right]^2 + 3\left[\mathrm{Im}\,(b)\right]^2} - a + \mathrm{Re}\,(b) \ , \qquad m_2 = m_0 \ , \\
m_3 &= \sqrt{\left[m_0 + a + 2\mathrm{Re}\,(b)\right]^2 + 3\left[\mathrm{Im}\,(b)\right]^2} + a - \mathrm{Re}\,(b) \ , \qquad (7)
\end{aligned}
$$

together with $\rho = \sigma = 0$. Comparing our results with the well-known standard parametrization,[5] we immediately obtain $\sin\theta_{12} = 1/\left(\sqrt{2+\cos\theta}\right)$, $\sin\theta_{23} = 1/\sqrt{2}$, and $\sin\theta_{13} = 2/\left[\sqrt{6}\sin(\theta/2)\right]$. The leptonic Jarlskog parameter $\mathcal{J}$, which is a rephasing-invariant measure of CP violation in neutrino oscillations,[6] reads $\mathcal{J} = \sin\theta/(6\sqrt{3})$. If $\theta = 0^\circ$ holds, the tri-bimaximal neutrino mixing pattern (with $\tan\theta_{12} = 1/\sqrt{2}$ or $\theta_{12} \approx 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$)[7] will be reproduced. One can see that the soft breaking of μ-τ symmetry leads to both $\theta_{13} \neq 0^\circ$ and $\mathcal{J} \neq 0$, but it does not affect the favorable result $\theta_{23} = 45^\circ$ given by the tri-bimaximal mixing pattern. On the other hand, $\sin\theta_{12} \approx 1/\sqrt{3}$ is an excellent approximation, since θ must be small to maintain the smallness of θ_{13}. In view of $\theta_{13} < 10^\circ$,[8] we obtain $\theta \lesssim 24.6^\circ$ and $\mathcal{J} \lesssim 0.04$. It is possible to measure $\mathcal{J} \sim \mathcal{O}(10^{-2})$ in the future long-baseline neutrino oscillation experiments. The neutrino masses in scenario (A) rely on four real model parameters m_0, a, Re(b) and Im(b). Thus it is easy to fit the neutrino mass-squared differences $\Delta m^2_{21} = (7.2\ldots 8.9) \times 10^{-5}$ eV^2 and $\Delta m^2_{32} = \pm(2.1\ldots 3.1) \times 10^{-3}$ eV^2.[8] Such a fit should not involve any fine-tuning, because (a) the number of free parameters is larger than the number of constraint conditions and (b) three neutrino masses have very weak correlation with three mixing angles. A detailed numerical analysis can be found in Ref. 4, and a remarkable feature is that only the normal neutrino mass hierarchy ($m_1 < m_2 < m_3$) is allowed in this scenario.

Scenario (B): a, b and c are all real, but m_0 is complex. By using Eqs. (3)-(6), we obtain $\tan\theta = \sqrt{3}\,(b-c)\,/(b+c-2a)$, and

$$
\begin{aligned}
m_1 &= \sqrt{\left[m_- + \mathrm{Re}\,(m_0)\right]^2 + \left[\mathrm{Im}\,(m_0)\right]^2} \ , \qquad m_2 = |m_0| \ , \\
m_3 &= \sqrt{\left[m_+ + \mathrm{Re}\,(m_0)\right]^2 + \left[\mathrm{Im}\,(m_0)\right]^2} \ , \qquad (8)
\end{aligned}
$$

where $m_\pm = (a+b+c) \pm \sqrt{a^2+b^2+c^2-ab-ac-bc}$. Two Majorana phases ρ

and σ are given by

$$\tan 2\rho = \frac{\left(m_+ - m_-\right) \mathrm{Im}\left(m_0\right)}{m_0^2 + m_+ m_- + \left(m_+ + m_-\right) \mathrm{Re}\left(m_0\right)} \; ,$$
$$\tan 2\sigma = \frac{m_+ \mathrm{Im}\left(m_0\right)}{m_0^2 + m_+ \mathrm{Re}\left(m_0\right)} \; . \tag{9}$$

Although ρ and σ have nothing to do with the behaviors of neutrino oscillations, they may significantly affect the neutrinoless double-beta decay.[9] Comparing our formulae with the standard parametrization, we arrive at $\sin\theta_{12} = 1/\sqrt{2+\cos\theta}$, $\sin\theta_{23} = \sqrt{2+\cos\theta-\sqrt{3}\sin\theta}/\sqrt{2\left(2+\cos\theta\right)}$, $\sin\theta_{13} = 2/\sqrt{6}\sin(\theta/2)$ together with $\delta = 0^\circ$ for the Dirac phase of CP violation. The results for θ_{12} and θ_{13} in this scenario are the same as those obtained in scenario (A), but the Jarlskog parameter $\mathcal{J}$ is now vanishing. Because of the μ-τ symmetry breaking, θ_{23} may somehow deviate from the favorable value $\theta_{23} = 45^\circ$. Given $\theta \lesssim 24.6^\circ$ corresponding to $\theta_{13} < 10^\circ$, θ_{23} is allowed to vary in the range $37.8^\circ \lesssim \theta_{23} \leq 45^\circ$. The neutrino masses depend on five real model parameters a, b, c, $\mathrm{Re}(m_0)$ and $\mathrm{Im}(m_0)$. Hence there is sufficient freedom to fit two observed neutrino mass-squared differences Δm^2_{21} and Δm^2_{32}. Our careful numerical analysis, which has been done in Ref. 4, shows that both normal ($m_1 < m_2 < m_3$) and inverted ($m_3 < m_1 < m_2$) neutrino mass hierarchies are allowed, and the Majorana phases (ρ, σ) are less restricted in scenario (B).

Although our discussions about the generalized FL model are restricted to low-energy scales, it can certainly be extended to a superhigh-energy scale (e.g., the GUT scale or the seesaw scale). In this case, one should take into account the radiative corrections to both neutrino masses and flavor mixing parameters when they run from the high scale to the electroweak scale.[10]

We conclude that the μ-τ symmetry and its slight breaking are useful and suggestive for model building. We expect that a stringent test of the generalized FL model, in particular its two simple and instructive scenarios, can be achieved in the near future from the neutrino oscillation and neutrinoless double-beta decay experiments.

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